

5.2 SPECIAL PRODUCTS & GROUPING

$$\begin{aligned} & \underbrace{(x+a)x} + \underbrace{(x+a)3} \\ & (x+a)(x+3) \end{aligned}$$

Ex. 1

$$\begin{aligned} \text{a.) } & (xy + 2y) + (5x + 10) \\ & y(x+2) + 5(x+2) \\ & (x+2)(y+5) \end{aligned}$$

$$\begin{aligned} \text{b.) } & (x^2 + wx) + (x + w) \\ & x(x+w) + 1(x+w) \\ & (x+w)(x+1) \end{aligned}$$

Ex. 2

$$\begin{aligned} \text{a.) } & (mn + 4m) + (m^2 + 4n) \\ & \underbrace{m(n+4)} + 1 \underbrace{(m^2 + 4n)} \end{aligned}$$

THESE TWO DON'T MATCH.
GO BACK AND REARRANGE.

$$\begin{aligned} & (mn + m^2) + (4m + 4n) \\ & m(n + m) + 4(m + n) \\ & (m+n)(m+4) \end{aligned}$$

$$\text{b.) } ax + b + bx + a$$

$$\begin{aligned} & (ax + a) + (bx + b) \\ & a(x+1) + b(x+1) \\ & (x+1)(a+b) \end{aligned}$$

$$\begin{aligned} & (ax + bx) + (a + b) \\ & x(a+b) + 1(a+b) \\ & (a+b)(x+1) \end{aligned}$$

Ex. 3

$$a.) \quad 2x^2 - 3x - 2x + 3$$

$$(2x^2 - 2x) + (-3x + 3)$$

$$2x(x-1) + -3(x-1)$$

$$(x-1)(2x-3)$$

$$b.) \quad ax + 3y - 3x - ay$$

$$(ax - ay) + (-3x + 3y)$$

$$a(x-y) + -3(x-y)$$

$$(x-y)(a-3)$$

DIFFERENCE OF SQUARES

$$a^2 - b^2 = (a + b)(a - b)$$

↑ ↑

BOTH OF THESE MUST BE PERFECT SQUARES

Ex. 4

a.) $y^2 - 81 = (y + 9)(y - 9)$

b.) $9m^2 - 16 = (3m + 4)(3m - 4)$

c.) $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$

PERFECT SQUARE TRINOMIAL

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex. 5

a.) $x^2 - 14x + 49 = (x-7)^2$

b.) $4x^2 - 81 = (2x+9)(2x-9)$

c.) $4a^2 + 24a + 25$ NOT A PERFECT SQUARE
NOT $20a$

d.) $9y^2 - 24y - 16$ NOT A PERFECT SQUARE
OR DIFF OF SQUARES

Ex. 6 $-2(2)x$

a.) $x^2 - 4x + 4 = (x - 2)^2$

b.) $a^2 + 16a + 64 = (a + 8)^2$

c.) $4x^2 - 12x + 9 = (2x - 3)^2$

Ex. 7 FACTOR COMPLETELY

a.) $2x^3 - 50x$

$$2x(x^2 - 25)$$

$$2x(x + 5)(x - 5)$$

b.) $8x^2y - 32xy + 32y$

$$8y(x^2 - 4x + 4)$$

$$8y(x - 2)^2$$

c.) $(2x^3 - 3x^2) + (-2x + 3)$

$$x^2(2x - 3) + -1(2x - 3)$$

$$(2x - 3)(x^2 - 1)$$

$$(2x - 3)(x + 1)(x - 1)$$

5.3 FACTORING $ax^2 + bx + c$ WHEN $a=1$

$$\cdot ax^2 + bx + c$$

LOOK FOR m & n SUCH THAT

$$m+n = b \quad \text{AND} \quad mn = c$$

$$(x + \underline{m})(x + \underline{n})$$

Ex.1

$$a.) x^2 + 5x + 6 = (x + 3)(x + 2)$$

$$b.) x^2 + 8x + 12 = (x + 6)(x + 2)$$

$$1, 12 \quad \textcircled{2, 6} \quad 3, 4$$

$$c.) a^2 - 9a + 20 = (a - 4)(a - 5)$$

$$-1, -20 \quad -2, -10 \quad \textcircled{-4, -5}$$

Ex. 2

a.) $x^2 + 5x + 4 = (x + 1)(x + 4)$

$(1, 4) \quad 2, 2$

b.) $y^2 + 6y - 16 = (y - 2)(y + 8)$

$-1, 16 \quad (-2, 8) \quad -4, 4$

c.) $w^2 - 5w - 24 = (w + 3)(w - 8)$

$1, -24 \quad 2, -12 \quad (3, -8) \quad 4, -6$

Ex.3

$$\begin{aligned} \text{a.) } 2x - 8 + x^2 &= x^2 + 2x - 8 \\ -1,8 \quad \textcircled{-2,4} &= (x-2)(x+4) \end{aligned}$$

$$\begin{aligned} \text{b.) } -36 + t^2 - 9t &= t^2 - 9t - 36 = (t+3)(t-12) \\ 1, -36 \quad 2, -18 \quad \textcircled{3, -12} \quad 4, -9 \quad 6, -6 \end{aligned}$$

Ex. 5

$$a.) x^2 + 2xy - 8y^2 = (x - 2y)(x + 4y)$$

$$b = 2y \quad c = -8y^2$$

$$-y, 8y, \quad (-2y, 4y)$$

$$b.) a^2 - 7ab + 10b^2 = (a - 2b)(a - 5b)$$

$$-1, -10 \quad (-2, -5)$$

$$c.) 1 - 2xy - 8x^2y^2 = (1 + 2xy)(1 - 4xy)$$

5.4 FACTORING $ax^2 + bx + c$ WITH $a \neq 1$

THE AC METHOD

$$ax^2 + bx + c$$

FIND ac

FIND TWO FACTORS

OF ac THAT ADD

UP TO b

REWRITE & FACTOR BY GROUPING

Ex. 1

a.) $2x^2 + 7x + 6$

$$(2x^2 + 4x) + (3x + 6)$$

$$2x(x+2) + 3(x+2)$$

$$(2x+3)(x+2)$$

$$ac = (2)(6) = 12$$

$$1 \quad 12$$

$$2 \quad 6$$

$$\begin{array}{|c|c|} \hline 3 & 4 \\ \hline \end{array}$$

Ex. 3

a.) $2x^2 + 5x - 3$

~~$(2x + 3)(x - 1)$~~

$(2x - 1)(x + 3)$

$$ax^2 + bx + c$$

$$(ax + m)(x + n)$$

$$an + m = b$$

$$mn = c$$