5.4 Ex. 5 Factor Completely

a.) $4x^3 + 14x^2 + 6x$

$2x(2x^2 + 7x + 3)$

$ac = (2)(3) = 6$

1 6

2 3

$2x[(2x^2 + 1x) + (6x + 3)]$

$2x[x(2x+1) + 3(2x+1)]$

$2x[(2x+1)(x+3)]$

$2x(2x+1)(x+3)$

b) $12x^2y + 6xy + 6y$

$6y[2x^2 + x + 1]$

$ac = (2)(1) = 2$

$1 \quad 2$

NOT FACTORABLE

5.5 Difference and Sum of Cubes

$$a^2 - b^2 = (a+b)(a-b)$$ "Difference of squares"

"Perfect square trinomials":

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$ "Difference of cubes"

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$ "Sum of cubes"

$$(a-b)(a^2+ab+b^2)$$

$$a^3+\cancel{a^2b}+\cancel{ab^2}-\cancel{a^2b}-\cancel{ab^2}-b^3$$

$$a^3-b^3$$

$$(a+b)(a^2-ab+b^2)$$

$$a^3-\cancel{a^2b}+\cancel{ab^2}+\cancel{a^2b}-\cancel{ab^2}+b^3$$

$$a^3+b^3$$

Ex. 1 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

a) $w^3 - 8 = (w - 2)(w^2 + 2w + 4)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

b.) $x^3 + 1 = (x + 1)(x^2 - x + 1)$

c.) $8y^3 - 27 = (2y - 3)(4y^2 + 6y + 9)$

Ex. 2

a.) $x^4 - 16 = x^4 - 2^4 = (x^2)^2 - (2^2)^2$

$= (x^2+4)(x^2-4)$

$x^4 - 4x^2 + 4x^2 - 16 = x^4 - 16$

$= (x^2+4)(x+2)(x-2)$

$a^4 - b^4 = (a^2+b^2)(a+b)(a-b)$

b.) $81m^4 - n^4 = (9m^2+n^2)(9m^2-n^2)$

$(9m^2+n^2)(3m-n)(3m+n)$

Ex. 3

a.) $2a^2b - 24ab + 72b$

$2b[a^2 - 12a + 36]$ $2b(a-6)(a-6)$

$2b(a-6)^2$

b) $3x^3 + 6x^2 - 75x - 150$

$3[(x^3 + 2x^2) + (-25x - 50)]$

$3[x^2(x+2) + -25(x+2)]$

$3(x+2)(x^2-25)$

$3(x+2)(x+5)(x-5)$

c.) $-3x^4 - 15x^3 + 72x^2$

$-3x^2[x^2 + 5x - 24]$

$-3x^2(x-3)(x+8)$

d.) $60y^3 - 85y^2 - 25y$

$5y[12y^2 - 17y - 5]$

$5y[(12y^2 + 3y) + (-20y - 5)]$

$5y[3y(4y+1) + -5(4y+1)]$

$5y(4y+1)(3y-5)$

$ac = (12)(-5) = -60$

1 -60
2 -30
3 -20
4 -15
5 -12
6 -10

5.6 Solving Quadratic Equations by Factoring

$$ax^2 + bx + c = 0$$ "General form of a quadratic eqn."

"Solving" means finding the x values that make this true.

Zero Factor Property

If $ab = 0$, then $a = 0$ or $b = 0$.

Ex. 1

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x+3=0$ $\quad$ $x-2=0$

$-3 \quad -3$ $\quad$ $+2 \quad +2$

$x = -3$ $\quad$ $x = 2$

$(-3)^2 + (-3) - 6 = 0$?

$9 - 3 - 6 = 0$?

$0 = 0$ ✓

$(2)^2 + (2) - 6 = 0$?

$4 + 2 - 6 = 0$?

$0 = 0$ ✓

Ex. 2

$3x^2 = -3x$

$+3x \quad +3x$

$3x^2 + 3x = 0$

$3x(x+1) = 0$

$\frac{3x}{3} = \frac{0}{3}$ $\qquad$ $x + 1 = 0$

$x = 0$ $\qquad$ $x = -1$

$3(0)(0+1) = 0$?

$0 = 0$ ✓

$3(-1)(-1+1) = 0$?

$-3(0) = 0$?

$0 = 0$ ✓

Ex. 3

$(2x+1)(x-1) = 14$

$2x^2 - x - 1 = 14$

$-14 \quad -14$

$2x^2 - x - 15 = 0$

$(2x^2 - 6x) + (5x - 15) = 0$

$2x(x-3) + 5(x-3) = 0$

$(2x+5)(x-3) = 0$

$ac = (2)(-15) = -30$

1	-30
2	-15
3	-10
5	-6

$2x + 5 = 0$

$-5 \quad -5$

$\frac{2x}{2} = \frac{-5}{2}$

$x = -\frac{5}{2}$

$x - 3 = 0$

$x = 3$

Ex. 4

$$5x^2 - 30x + 45 = 0$$

$$\frac{5(x^2 - 6x + 9)}{5} = \frac{0}{5}$$

$$x^2 - 6x + 9 = 0$$

$$\sqrt{(x-3)^2} = \sqrt{0}$$

$$x - 3 = 0$$

$$x = 3$$

Ex 5

$$(2x^3 - x^2) + (-8x + 4) = 0$$

$$x^2(2x-1) + -4(2x-1) = 0$$

$$(2x-1)(x^2-4) = 0$$

$$(2x-1)(x+2)(x-2) = 0$$

$2x-1=0$ $\quad x+2=0$ $\quad x-2=0$

$x = \frac{1}{2}$ $\quad x = -2$ $\quad x = 2$

Ex. 6

a.) $\left[\frac{1}{12}x^2 + \frac{1}{6}x - 2 = 0\right] 12$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6 \quad x = 4$

b.) $\left[0.02x^2 - 0.19x - 0.1 = 0\right] 100$

$2x^2 - 19x - 10 = 0$