CHAPTER 6: RATIONAL EXPRESSIONS

RATIONAL NUMBER: $\dfrac{a}{b}$

CAN BE WRITTEN AS A FRACTION

OF TWO INTEGERS

RATIONAL EXPRESSION:

ALGEBRAIC EXPRESSION THAT IS

A FRACTION OF TWO ALGEBRAIC EXPRESSIONS

$\dfrac{ax+b}{cx+d}$ IS AN EXAMPLE

$\underline{\text{Ex. 1}}$ EVALUATING RATIONAL EXPRESSIONS

a) $\dfrac{4x-1}{x+2}$ WHEN $x=-3$

$$\frac{4(-3)-1}{(-3)+2} = \frac{-13}{-1} = 13$$

b) $\underbrace{R(x)}_{\text{FUNCTION NOTATION}} = \dfrac{3x+2}{2x-1}$ FIND $R(4)$

$$R(4) = \frac{3(4)+2}{2(4)-1} = \frac{14}{7} = 2$$

Ex. 2 Ruling out x values

a.) $\dfrac{x^2-1}{\underbrace{x+8}}$ $\quad \dfrac{(-8)^2-1}{(-8)+8} = \dfrac{64-1}{0} = \dfrac{63}{0}$ UNDEFINED

DOES NOT EXIST AT $x=-8$

* RATIONAL EXPRESSIONS DO NOT EXIST AT ANY VALUE OF THE VARIABLE THAT MAKES THE DENOMINATOR ZERO.

b) $\dfrac{x+2}{2x+1}$

TO FIND x VALUES WHERE THE EXPRESSION DNE, SET THE DENOMINATOR EQUAL TO ZERO & SOLVE

$$2x+\cancel{1}=0$$
$$\quad -\cancel{1} \quad -1$$
$$\frac{\cancel{2}x}{\cancel{2}}=\frac{-1}{2}$$

$x=-\frac{1}{2}$

c.) $\dfrac{x+5}{x^2-4}$

$$x^2-\cancel{4}=0$$
$$\quad +\cancel{4} \quad +4$$
$$\sqrt{x^2}=\sqrt{4}$$
$$x=\pm 2$$

THERE CAN BE MULTIPLE PLACES WHERE THE EXPRESSION IS UNDEFINED.

DOMAIN: SET OF VALUES OF THE VARIABLE FOR WHICH THE EXPRESSION IS DEFINED

Ex.3 FIND THE DOMAIN

a) $\dfrac{x^2-9}{x+3}$ $\rightarrow$ $x + \cancel{3} = 0$

$\quad -\cancel{3} \quad -3$

$x = -3$

SET NOTATION DOMAIN: $\{x \mid x \neq -3\}$

INTERVAL NOTATION DOMAIN: $(-\infty, -3) \cup (-3, \infty)$

b) $\dfrac{x}{x^2-x-6}$ $\rightarrow$ $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\dfrac{3}{3^2-3-6} = \dfrac{3}{0}$ $\qquad x - 3 = 0 \qquad x + 2 = 0$

$x = 3 \qquad x = -2$

$\dfrac{-2}{(-2)^2+2-6} = \dfrac{-2}{0}$

DOMAIN: $\{x \mid x \neq -2 \text{ AND } x \neq 3\}$ SET

$(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$ INTERVAL

c) $\dfrac{x-5}{4}$ $\rightarrow$ $4 \neq 0$

$\{x\}$ $\qquad \mathbb{R}$

DOMAIN: $(-\infty, \infty)$

REDUCING FRACTIONS:

IF $a \neq 0$ AND $c \neq 0$, THEN

$$\frac{ab}{ac} = \frac{b}{c}$$

Ex. 4

a) $\frac{30}{42} = \frac{\cancel{6}\cdot 5}{\cancel{6}\cdot 7} = \frac{5}{7}$

b) $\frac{x^2-9}{6x+18} = \frac{(x-3)\cancel{(x+3)}}{6\cancel{(x+3)}} = \frac{x-3}{6}$

c) $\frac{3x^2+9x+6}{2x^2-8} = \frac{3(x^2+3x+2)}{2(x^2-4)} = \frac{3(x+1)\cancel{(x+2)}}{2(x-2)\cancel{(x+2)}}$

$= \frac{3(x+1)}{2(x-2)}$

QUOTIENT RULE

IF $a \neq 0$ AND $m, n \in \mathbb{Z}$, THEN

$$\frac{a^m}{a^n} = a^{m-n}$$

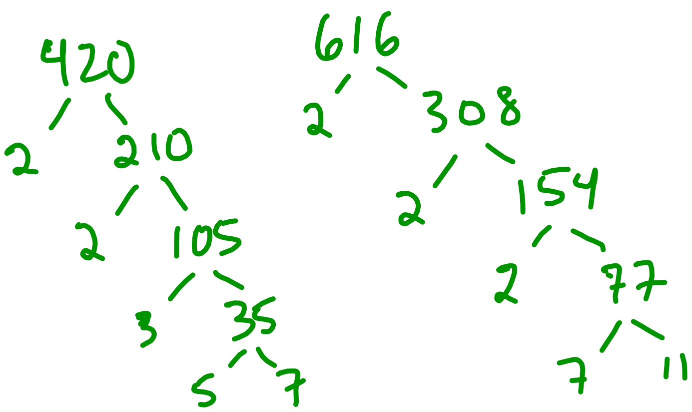
Ex.5 MORE REDUCING

a.) $\dfrac{3a^{15}}{6a^7} = \dfrac{a^8}{2}$

b) $\dfrac{6x^4y^2}{4xy^5} = \dfrac{3x^3}{2y^3}$

Ex.6 REDUCING WITH PRIME FACTORIZATION

$$\frac{420}{616} = \frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot 5\cdot \cancel{7}}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot \cancel{7}\cdot 11} = \frac{15}{22}$$



$$\frac{a-b}{b-a} = -1$$

Ex. 7

a) $$\frac{5x-5y}{4y-4x} = \frac{-5\cancel{(x-y)}}{4\cancel{(y-x)}} = -\frac{5}{4}$$

b) $$\frac{m^2-n^2}{n-m} = \frac{-(m+n)\cancel{(m-n)}}{\cancel{(n-m)}} = -(m+n)$$

$$= -m-n$$

Ex. 8

$$\frac{-3w-3w^2}{w^2-1} = \frac{-3w\cancel{(1+w)}}{\cancel{(w+1)}(w-1)} = \frac{-3w}{w-1}$$

Ex. 9

a.) 500mi in $x+1$ hrs: SPEED $= \dfrac{500}{x+1}$

b.) 100lbs FOR $\$x$: PRICE PER POUND $= \dfrac{x}{100}$

c.) 1 HOUSE PAINTED IN $2x$ HRS: RATE $= \dfrac{1}{2x}$

MULTIPLYING & DIVIDING RATIONALS

FOR $b \neq 0,\ c \neq 0,\ d \neq 0$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Ex.1

$$\frac{6}{7} \cdot \frac{14}{15} = \frac{84}{105} = \frac{4}{5}$$

$$\frac{6}{7} \cdot \frac{14}{15} = \frac{2 \cdot \cancel{3} \cdot 2 \cdot \cancel{7}}{\cancel{7} \cdot \cancel{3} \cdot 5} = \frac{4}{5}$$

Ex. 2

a.) $$\frac{9x}{5y}\cdot\frac{10y}{3xy}=\frac{2\cdot3\cdot\cancel{3\cdot5}\cdot\cancel{x}\cancel{y}}{\cancel{3\cdot5}\cdot\cancel{x}\cdot\cancel{y}\cdot y}=\frac{6}{y}$$

b.) $$\frac{-8xy^4}{3z^3}\cdot\frac{15z}{2x^5y^3}=\frac{-2\cdot2\cdot\cancel{2}\cdot\cancel{3}\cdot5\cdot\cancel{x}\cdot y^{\cancel{4}}\cdot\cancel{z}}{\cancel{2}\cdot\cancel{3}\cdot x^{\cancel{5}4}\,\cancel{y^{3}}\,z^{\cancel{3}2}}$$

$$=\frac{-20y}{x^4z^2}$$

Ex. 3

a.) $\dfrac{2x-2y}{4} \cdot \dfrac{2x}{x^2-y^2} = \dfrac{\cancel{2} \cdot \cancel{2}x\,\cancel{(x-y)}}{\cancel{2} \cdot \cancel{2}(x+y)\cancel{(x-y)}} = \dfrac{x}{(x+y)}$

b.) $\dfrac{x^2+7x+12}{2x+6} \cdot \dfrac{x}{x^2-16} = \dfrac{\cancel{(x+3)}\cancel{(x+4)}\,x}{2\cancel{(x+3)}\cancel{(x+4)}(x-4)} = \dfrac{x}{2(x-4)}$

c.) $\dfrac{a+b}{6a} \cdot \dfrac{8a^2}{a^2+2ab+b^2} = \dfrac{4\,\cancel{8}a^{\cancel{2}}\cancel{(a+b)}}{3\,\cancel{6}\cancel{a}\,(a+b)^{\cancel{2}}} = \dfrac{4a}{3(a+b)}$

$$\frac{2 \cdot 2 \cdot \cancel{2} \cdot \cancel{a} \cdot a\,\cancel{(a+b)}}{\cancel{2} \cdot 3 \cdot \cancel{a}\,\cancel{(a+b)}(a+b)} = \frac{4a}{3(a+b)}$$

Ex.5

a) $\frac{5}{3x} \div \frac{5}{6x} = \frac{5}{3x} \cdot \frac{6x}{5} = \frac{2 \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{x}}{\cancel{3} \cdot \cancel{5} \cdot \cancel{x}} = 2$

b) $\frac{x^7}{2} \div \frac{(2x^2)}{1} = \frac{x^7}{2} \cdot \frac{1}{2x^2} = \frac{x^7}{2 \cdot 2 \cdot x^2} = \frac{x^5}{4}$

c) $\frac{4-x^2}{x^2+x} \div \frac{x-2}{x^2-1} = \frac{\cancel{(2-x)}(2+x)\cancel{(x+1)}(x-1)}{-x\cancel{(x+1)}\cancel{(x-2)}}$

$= - \frac{(x+2)(x-1)}{x} = \frac{(1-x)(2+x)}{x}$

Ex. 6

a.) $$\frac{\frac{a+b}{3}}{\frac{1}{6}} = \frac{a+b}{3} \cdot \frac{6}{1} = 2(a+b)$$

b.) $$\frac{\frac{x^2-1}{2}}{\frac{x-1}{3}} = \frac{(x+1)(x-1)}{2} \cdot \frac{3}{x-1} = \frac{3(x+1)}{2}$$

c.) $$\frac{\frac{a^2+5}{3}}{2} = \frac{a^2+5}{3} \cdot \frac{1}{2} = \frac{a^2+5}{6}$$